

# **SOME THEORY BEHIND REAL-TIME RENDERING**



**Jaroslav Křivánek**

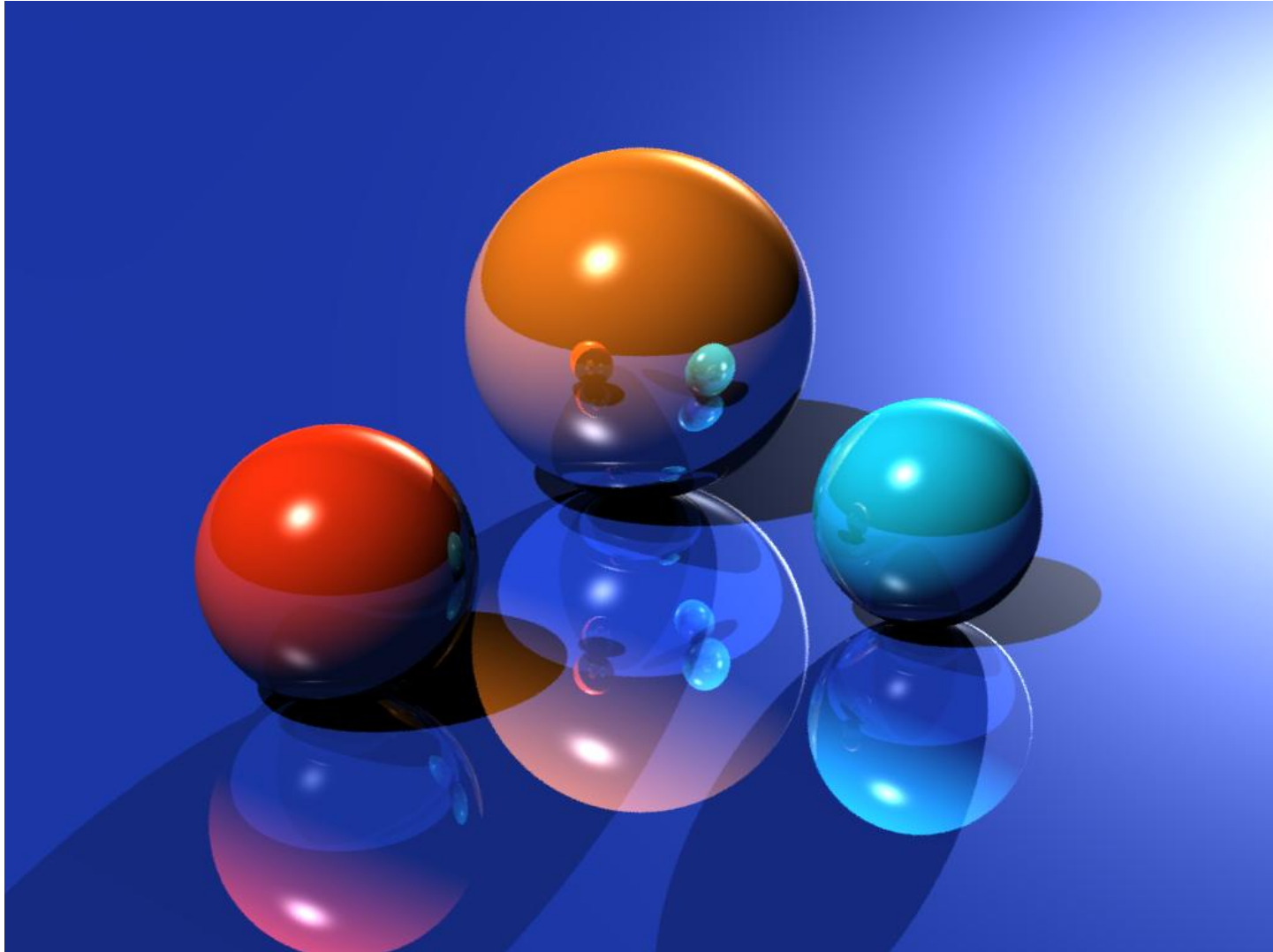
Charles University in Prague

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# **Off-line realistic rendering (not yet in rea-time)**

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# Ray tracing





corona







Image created by *Bertrand Benoit*  
Rendered in *Corona Renderer*



Image created by *Jeff Patton*  
Rendered in *Corona Renderer*







01 EKTORP three-seat sofa  
\$749

## A room with a view

Put a rocking chair in front of your favourite window and experience how relaxing it is to get away from it all by just coming home. Life is in full swing outside, but you feel totally calm.

**IKEA**® Seat cushions filled with high resilience foam provide comfortable support for your body when you sit. Cover: 53% linen, 47% viscose/crepe, 100% polyester. Risene natural.

**02 New FABRIKÖR glass-door cabinet \$399** The shelves in the cabinet are adjustable – makes it easy to adjust the height to suit what you want to store. May be completed with DÖDER LED lighting strip. Powder coated steel and tempered glass. Designer: Nike Karlsson. W57x D47, H150cm. Light green 702.422.94

**03 VÄRMÖÖ rocking-chair \$169** Wooden furniture that is suitable for both indoor and outdoor use. Solid pine. Designer: Nike Karlsson. W65x D74, H106cm. Black 002.059.59

**04 BJÖRNLOKA rug, flatwoven \$199** The durable, soil-resistant wool surface makes this rug perfect in your living room or under your dining table. The rug is machine-woven. User surface: 100% pure new wool. W170x L240cm. Beige/black 402.290.05

**05 HEMNES coffee table \$229** Stained, clear lacquered solid pine. Designer: Carina Bengt. L90x W90, H46cm. Grey-brown 402.579.51



NEW LOWER PRICE  
03 VÄRMÖÖ rocking-chair ~~\$299~~  
\$169

04 BJÖRNLOKA rug, flatwoven  
\$199





Image created by *Weta Digital*  
© 20<sup>th</sup> Century Fox



Image created by *Weta Digital*  
© 20<sup>th</sup> Century Fox





vimeo >> "The Great Gatsby VFX"



vimeo >> "The Great Gatsby VFX"



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# **Global illumination**

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# Global illumination – Color bleeding





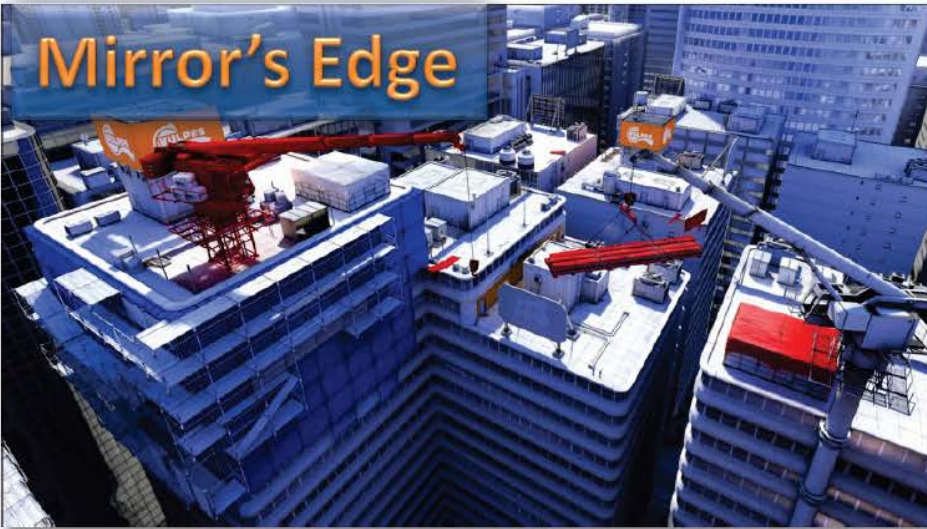
# Global illumination – Caustics





# Global illumination in games

Mirror's Edge



Halo 3



RAGE



Danger Planet



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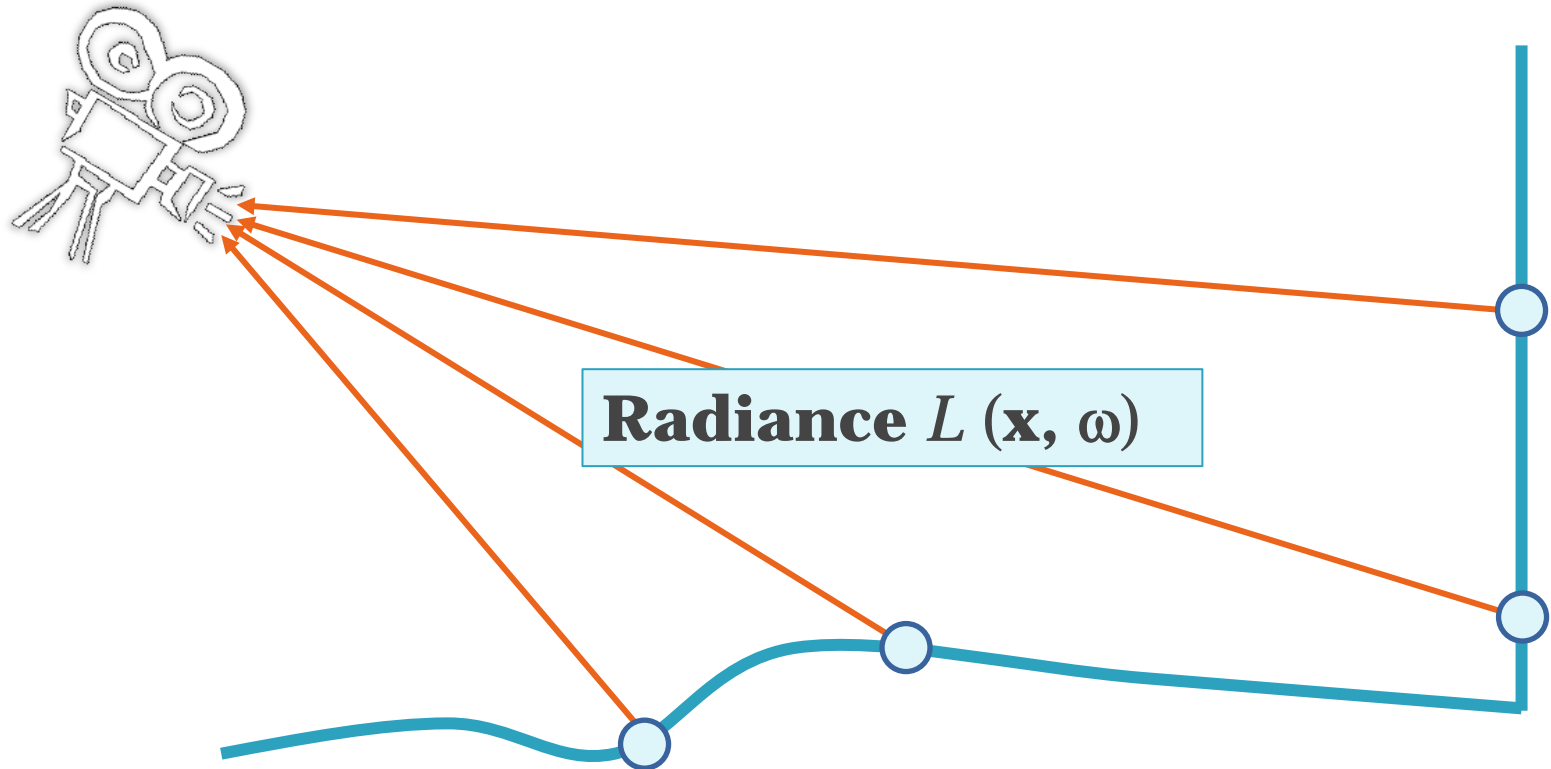
# **Basic light transport theory**

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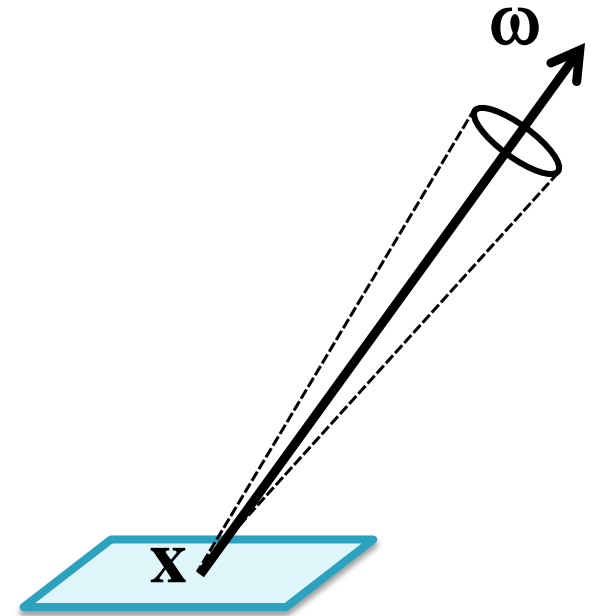
# Image generation

- **Pixel value** = average **radiance** reflected from surfaces visible through the pixel
- Generating an image involves (some sort of) **light transport simulation**

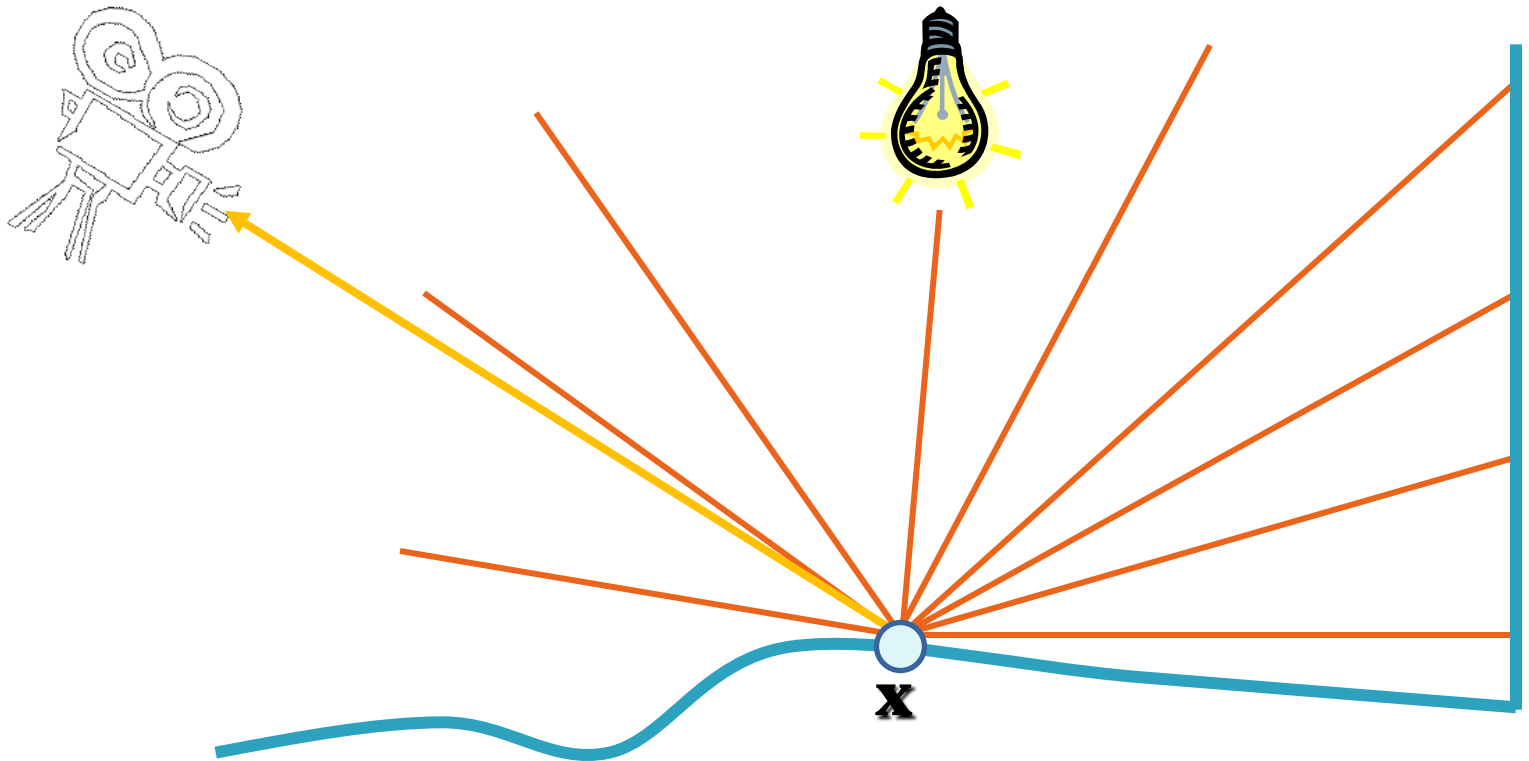


# Radiance

- Radiometric quantity measuring “amount of light energy” along a ray
- Denoted  $L(\mathbf{x}, \omega)$ 
  - $\mathbf{x}$  ... position
  - $\omega$  ... direction
- Units [W / m<sup>2</sup> sr ]
- Essential properties of radiance
  - Proportional to perceived brightness
    - XXXXX
  - Constant along a ray
    - XXXXXX



# Light reflection

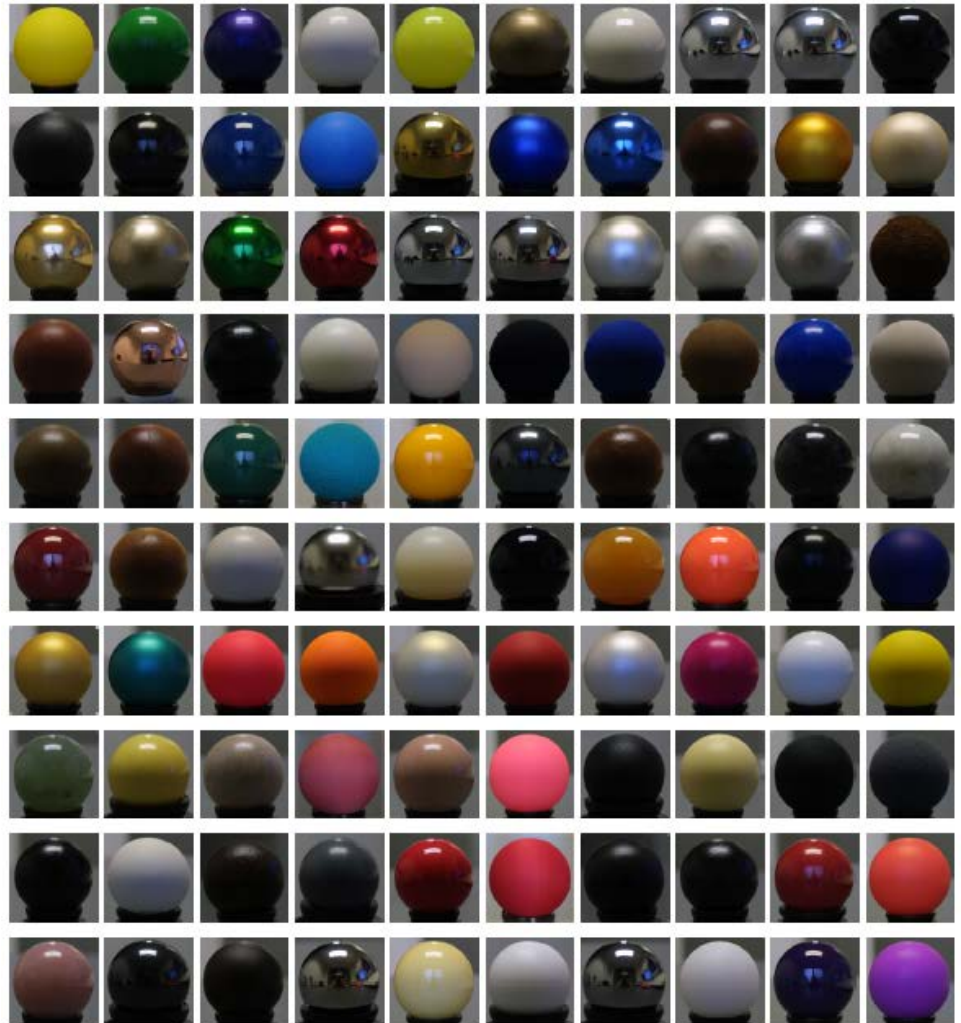




# Light reflection

- **BRDF**

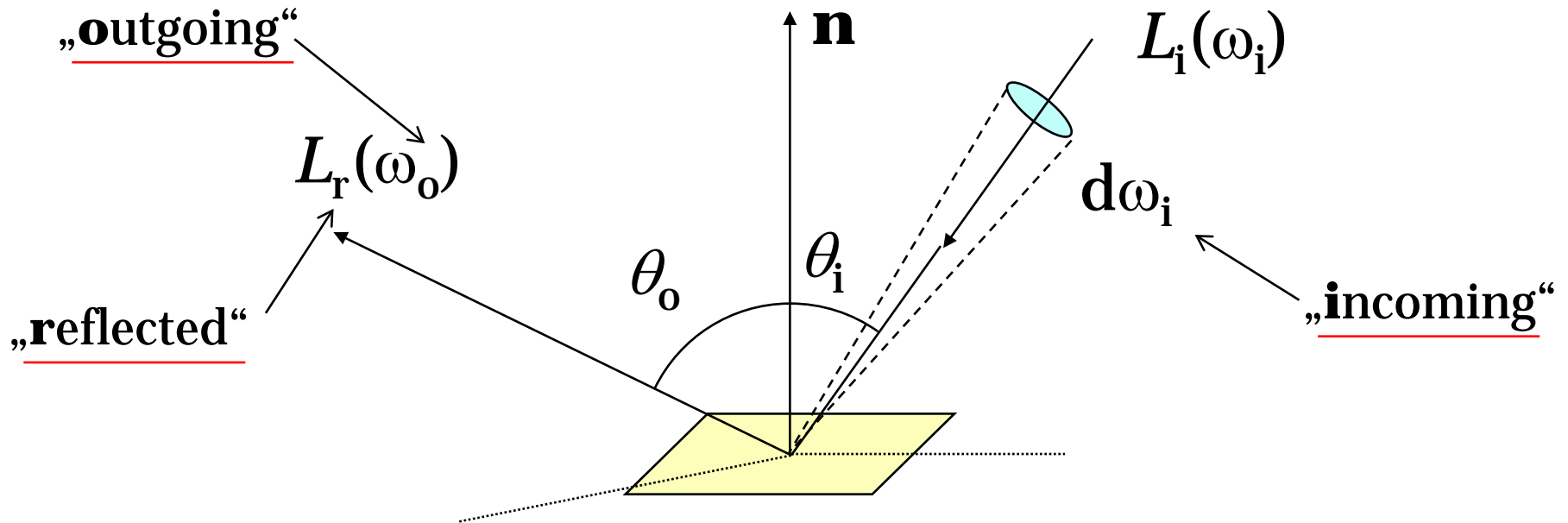
- Bi-directional  
Reflectance  
Distribution  
Function



Obr. Wojciech Matusik

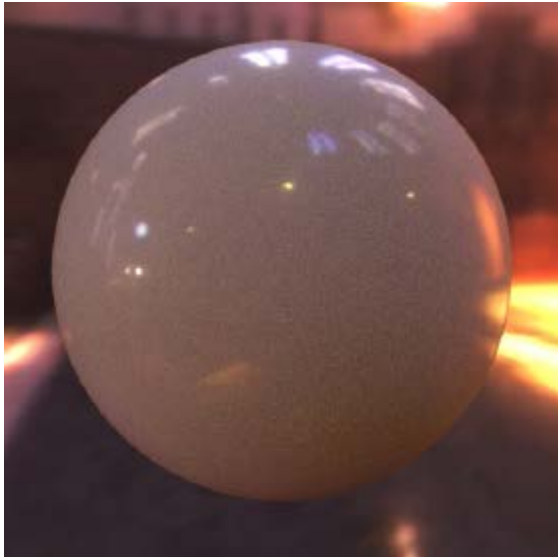
# BRDF – Formal definition

- **Bidirectional Reflectance Distribution Function**

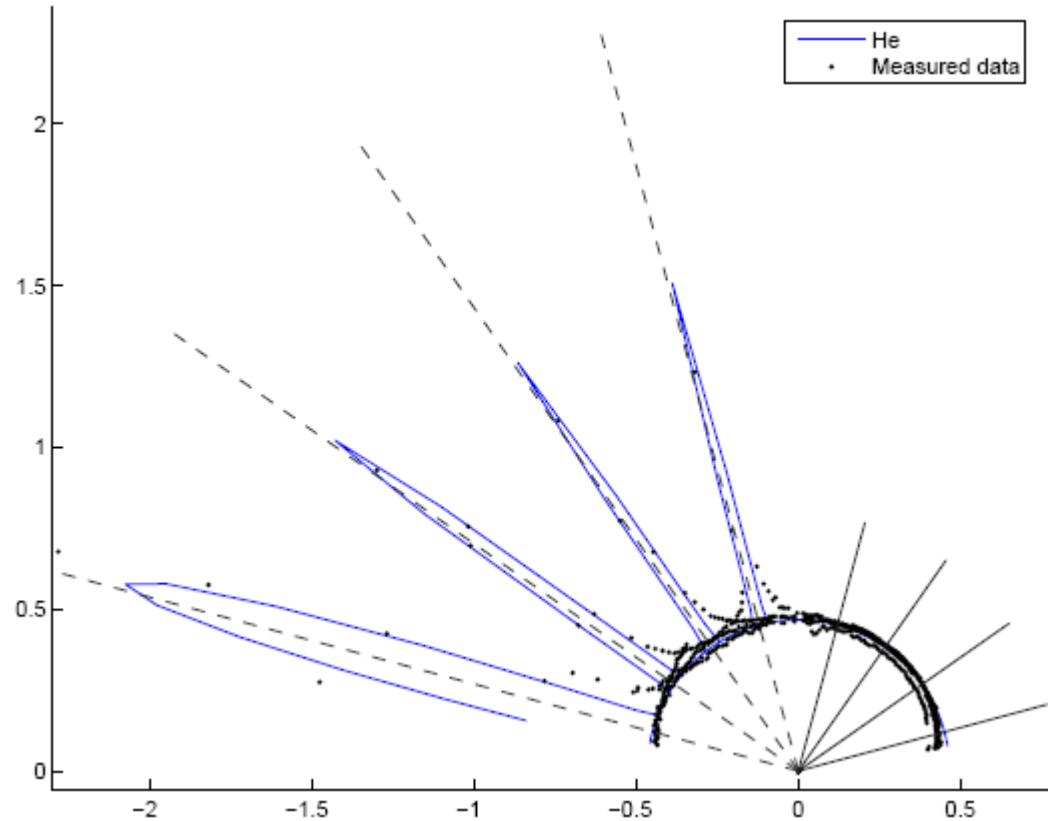


$$f_r(\omega_i \rightarrow \omega_o) = \frac{dL_r(\omega_o)}{L_i(\omega_i) \cdot \cos \theta_i \cdot d\omega_i} \quad [\text{sr}^{-1}]$$

# Surface appearance and the BRDF



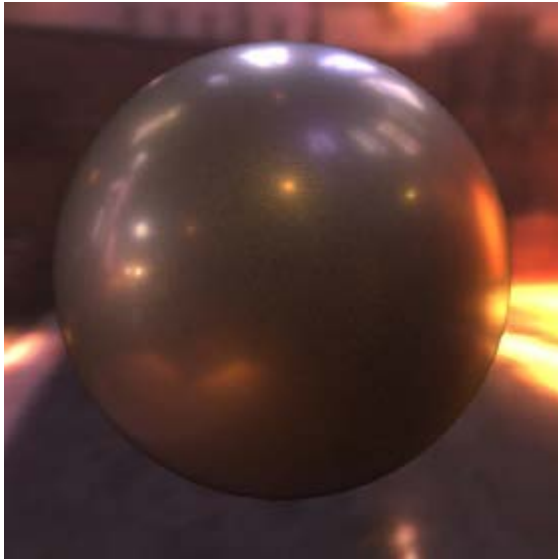
Appearance



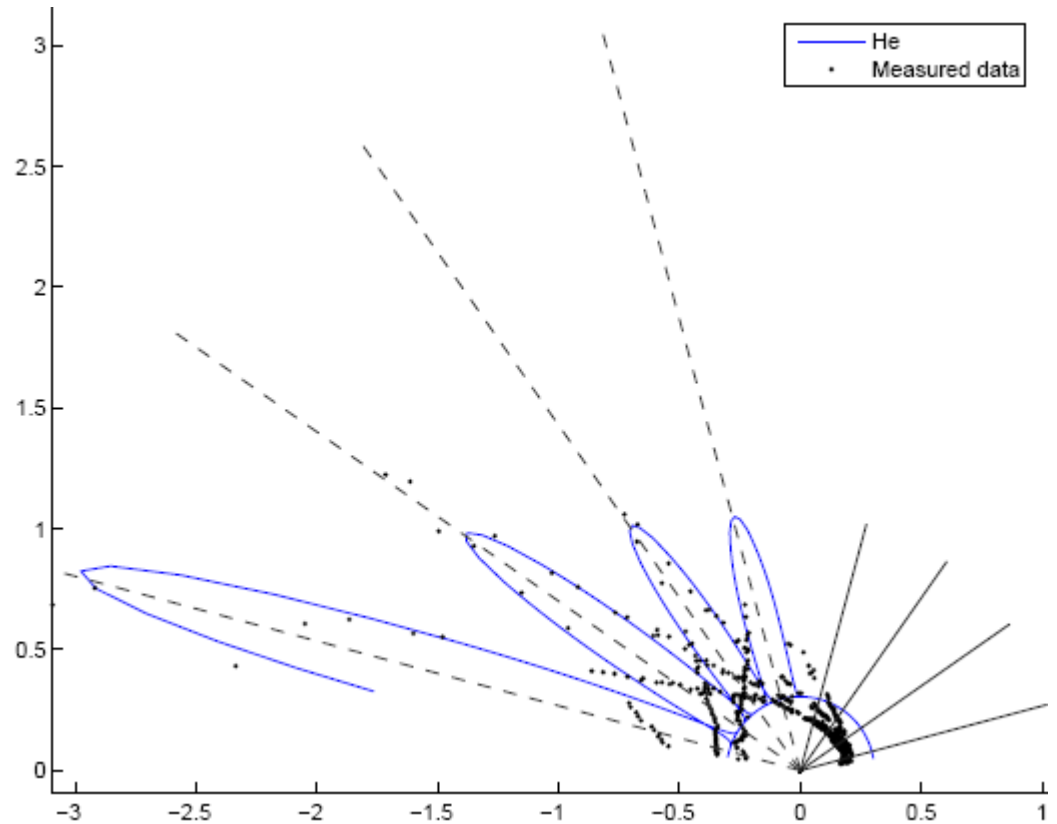
BRDF lobe  
(for four different viewing directions)



# Surface appearance and the BRDF

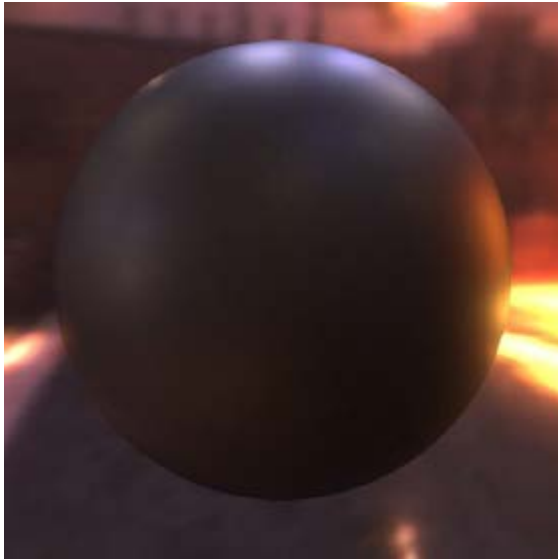


Appearance

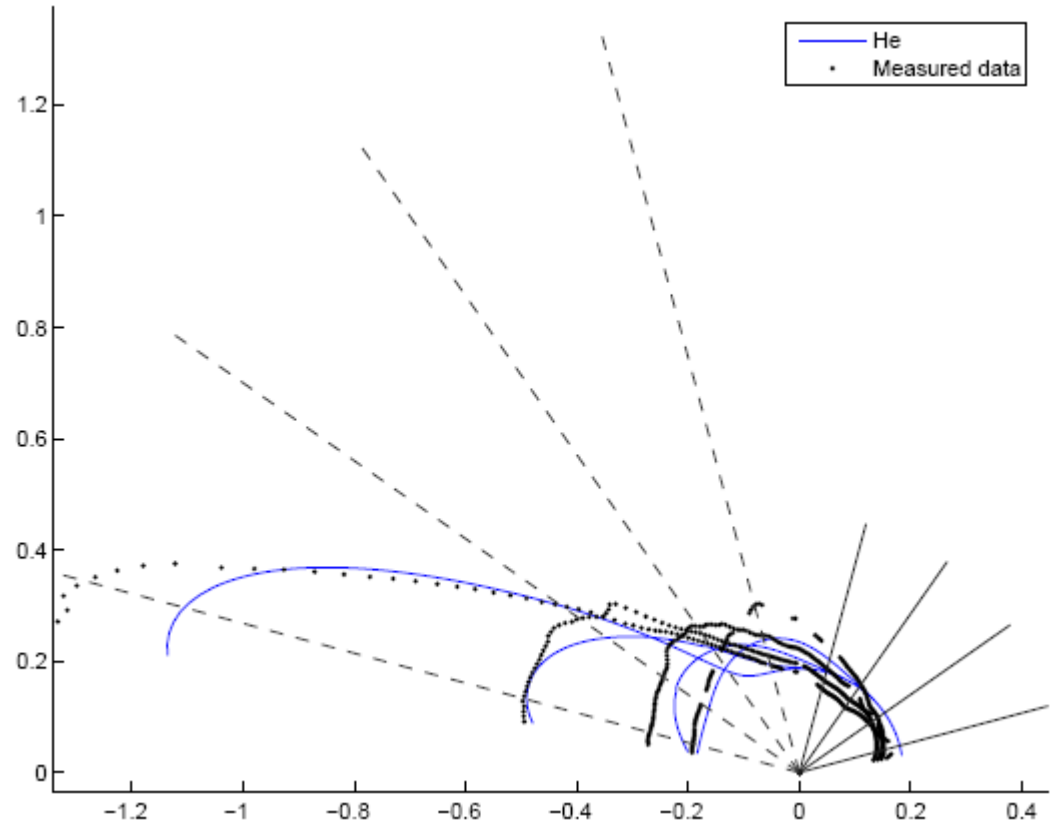


BRDF lobe  
(for four different viewing directions)

# Surface appearance and the BRDF

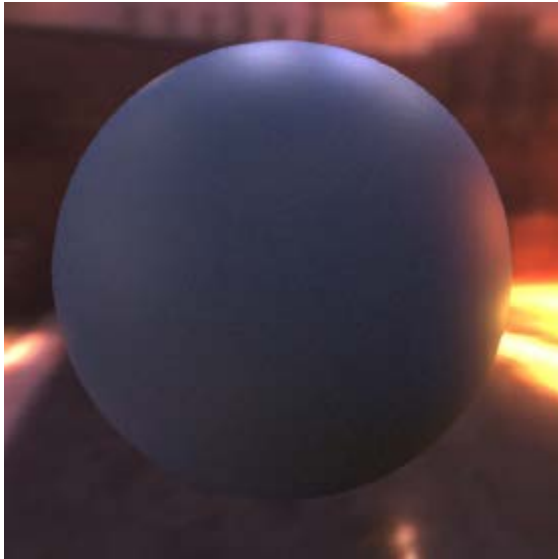


Appearance

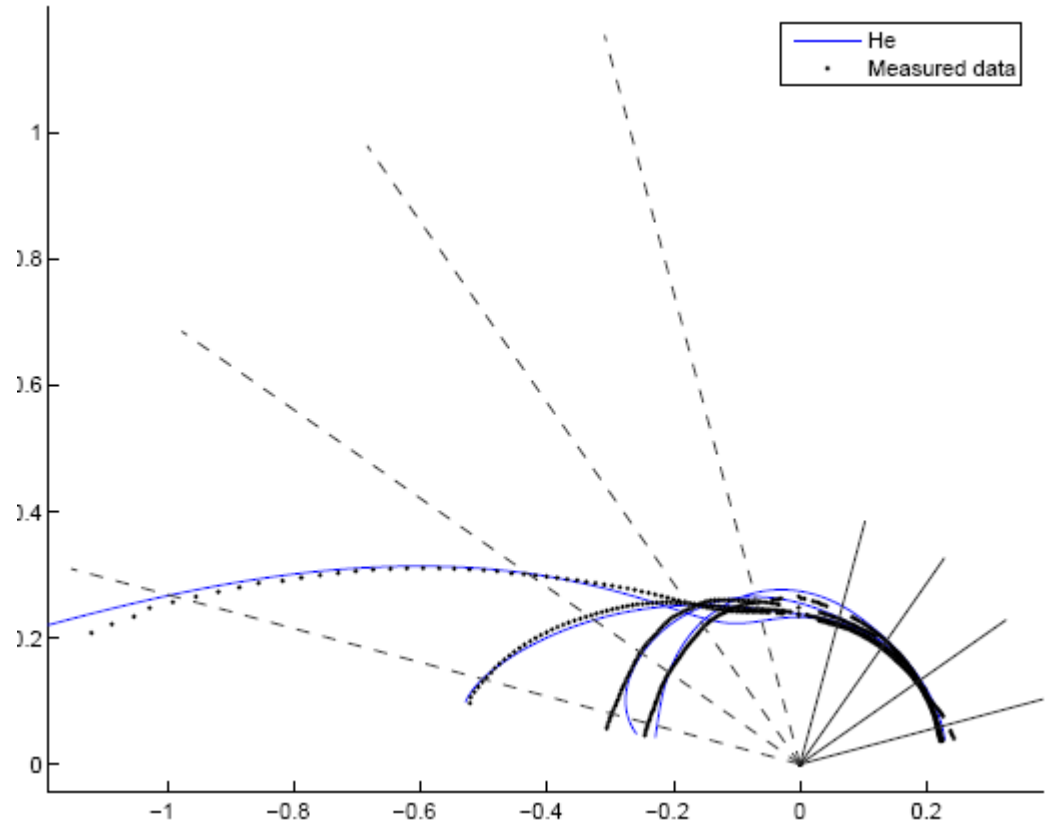


BRDF lobe  
(for four different viewing directions)

# Surface appearance and the BRDF



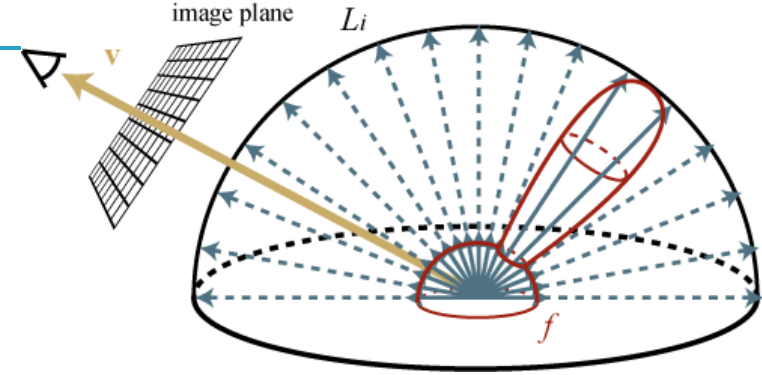
Appearance



BRDF lobe  
(for four different viewing directions)



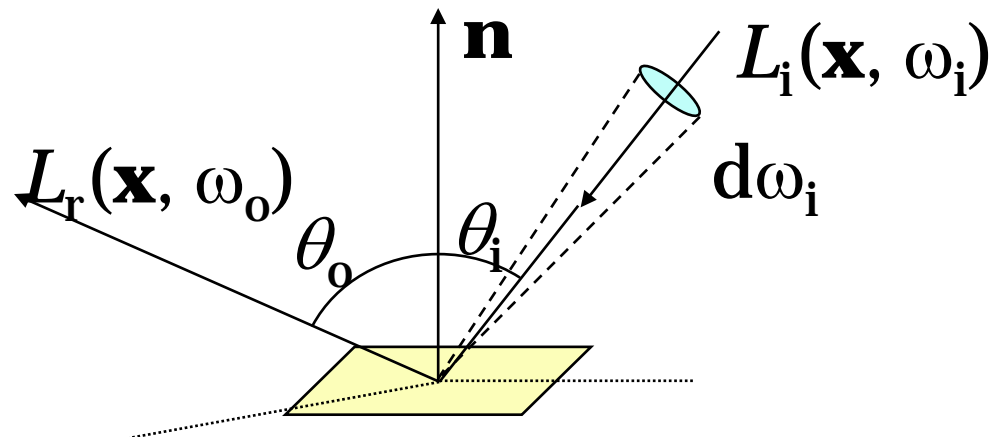
# Reflection equation



- Total reflected radiance: integrate contributions of incident radiance, weighted by the BRDF, over the hemisphere

$$L_r(\mathbf{x}, \omega_o) = \int_{H(\mathbf{x})} L_i(\mathbf{x}, \omega_i) \cdot f_r(\mathbf{x}, \omega_i \rightarrow \omega_o) \cdot \cos \theta_i \, d\omega_i$$

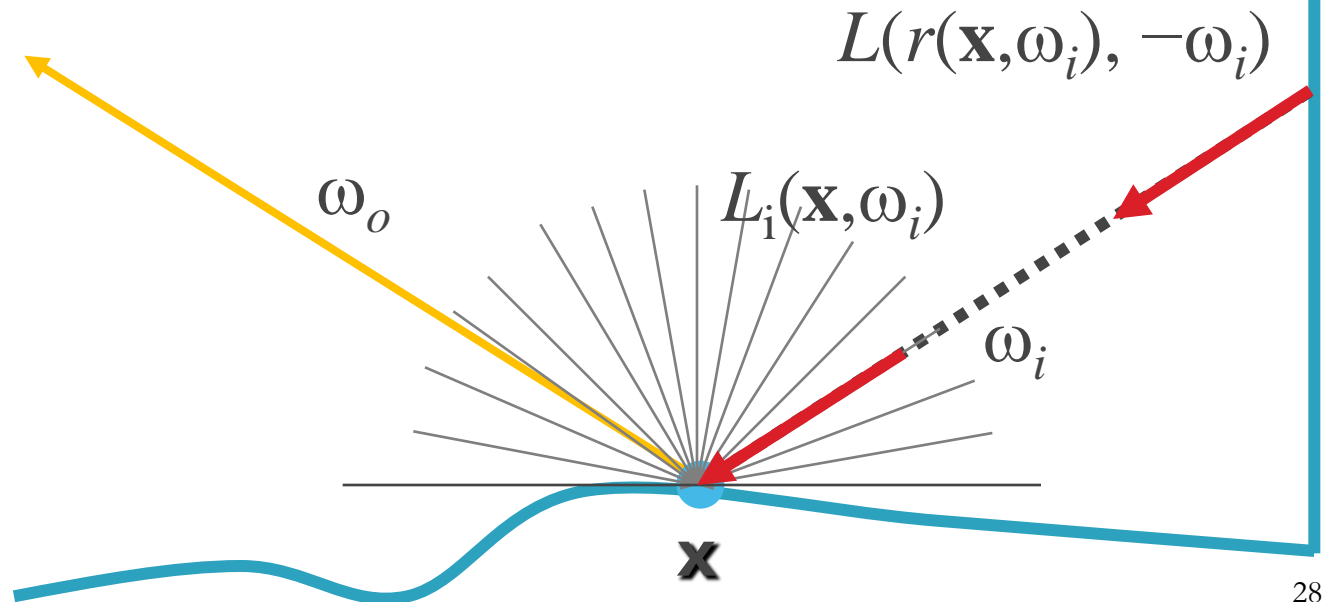
upper hemisphere  
over  $\mathbf{x}$



# Rendering Equation

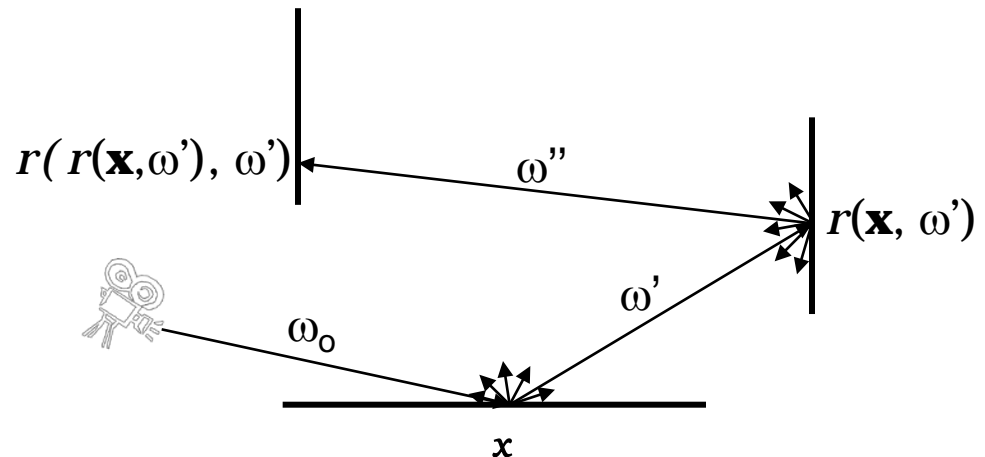
- But where does the incident radiance come from? Other surfaces in the scene!

$$L_o(\mathbf{x}, \omega_o) = L_e(\mathbf{x}, \omega_o) + \int_{H(\mathbf{x})} L_o(\mathbf{r}(\mathbf{x}, \omega_i), -\omega_i) \cdot f_r(\mathbf{x}, \omega_i \rightarrow \omega_o) \cdot \cos \theta_i \, d\omega_i$$



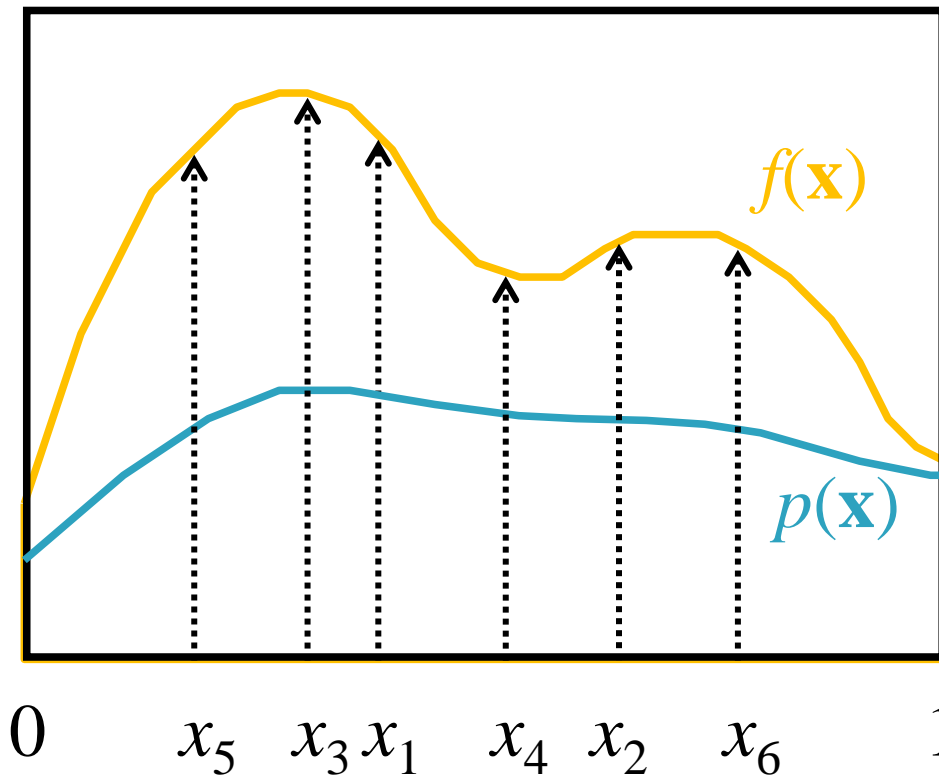
# Recursive calculation of rendering equation

- To calculate  $L(\mathbf{x}, \omega_0)$  I need to calculate  $L(r(\mathbf{x}, \omega'), -\omega')$  for all directions  $\omega'$  around the point  $\mathbf{x}$ .
- For the calculation of each  $L(r(\mathbf{x}, \omega'), -\omega')$  I need to do the same thing recursively
- At each step, apply Monte Carlo integration to sample the hemisphere
- etc.



# Monte Carlo integration

- General approach to numerical evaluation of integrals



Integral:

$$I = \int f(x) dx$$

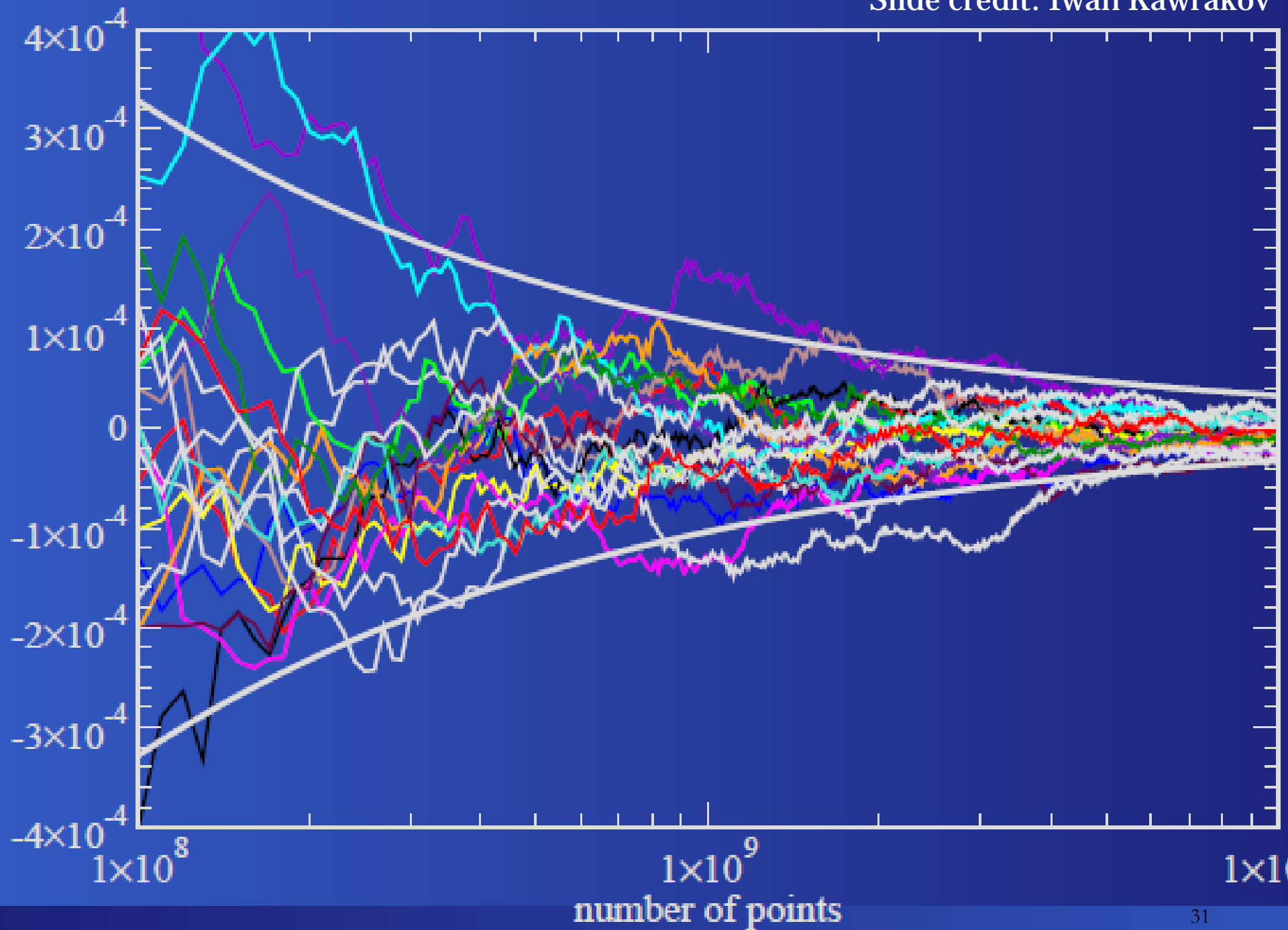
Monte Carlo estimate of  $I$ :

$$\langle I \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f(x_i)}{p(x_i)}; \quad x_i \propto p(x)$$

Correct „on average“:

$$E[\langle I \rangle] = I$$





# Rendering equation – Operator form

$$L = L_e + T \circ L$$



- $L$ : Equilibrium radiance
- $L_e$ : Emitted radiance
- $T$ : Transport & scattering operator

# Progressive approximation

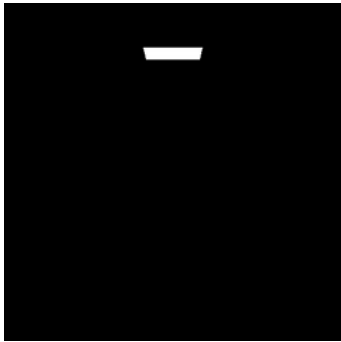
- Each application of  $T$  corresponds to one step of reflection & light propagation

$$L = L_e + TL_e + T^2 L_e + T^3 L_e + \dots$$

The diagram shows the equation  $L = L_e + TL_e + T^2 L_e + T^3 L_e + \dots$ . A green box highlights the first two terms,  $L_e + TL_e$ . Arrows point from labels below to terms in the equation: 'emission' points to  $L_e$ ; 'direct illumination' points to  $TL_e$ ; 'one-bounce indirect illumination' points to  $T^2 L_e$ ; and 'two-bounce indirect illumination' points to  $T^3 L_e$ .

OpenGL shading

# Progressive approximation



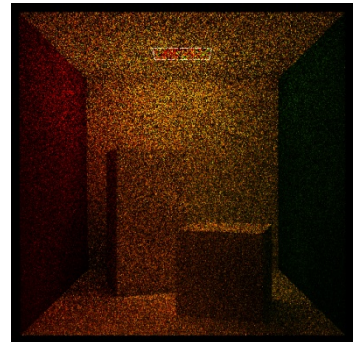
$L_e$



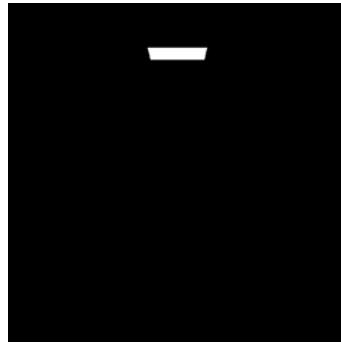
$T \circ L_e$



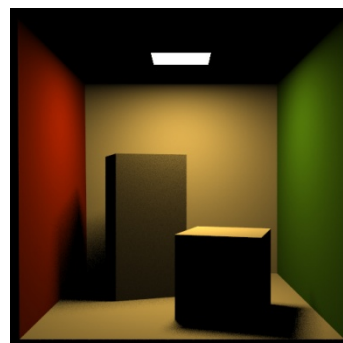
$T \circ T \circ L_e$



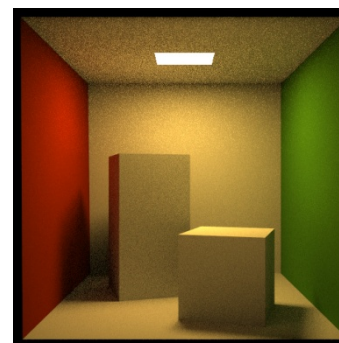
$T \circ T \circ T \circ L_e$



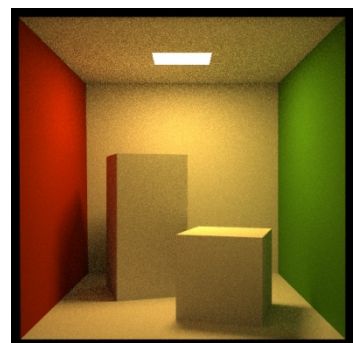
$L_e$



$L_e + T \circ L_e$



$L_e + T L_e + T^2 L_e$



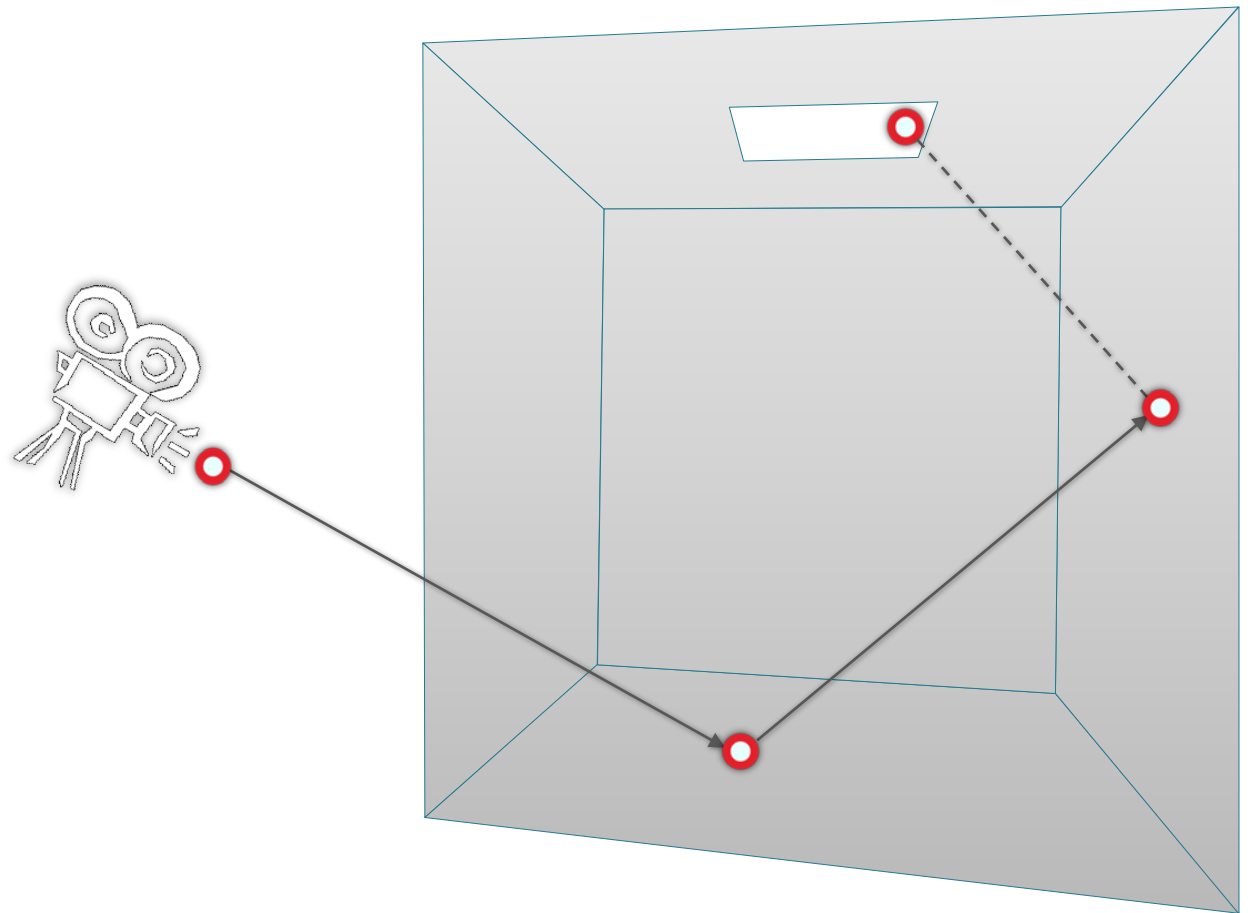
$L_e + \dots + T^3 L_e$



# Idea: Distributed ray tracing (Cook '84)



# Path tracing



# Path tracing, Naive

**getLi (x,  $\omega$ ):**

$\mathbf{y} = \text{traceRay}(\mathbf{x}, \omega)$

return

$\text{Le}(\mathbf{y}, -\omega) +$

// emitted radiance

$\text{Lr}(\mathbf{y}, -\omega)$

// reflected radiance

**Lr(y,  $\omega$ ):**

$\omega' = \text{genUniformRandomDir}(\mathbf{n}(\mathbf{y}))$

return  $\text{getLi}(\mathbf{y}, \omega') * \text{brdf}(\mathbf{y}, \omega, \omega') * \text{dot}(\omega', \mathbf{n}(\mathbf{y})) * 2\pi$

# Path tracing, Naive

**getLi (x,  $\omega$ ):**

$\mathbf{y} = \text{traceRay}(\mathbf{x}, \omega)$

return

$\text{Le}(\mathbf{y}, -\omega) +$

// emitted radiance

$\text{Lr}(\mathbf{y}, -\omega)$

// reflected radiance

**Lr(y,  $\omega$ ):**

$\omega' = \text{genUniformRandomDir}(\mathbf{n}(\mathbf{y}))$

return  $\text{getLi}(\mathbf{y}, \omega') * \text{brdf}(\mathbf{y}, \omega, \omega') * \text{dot}(\omega', \mathbf{n}(\mathbf{y})) * 2\pi$

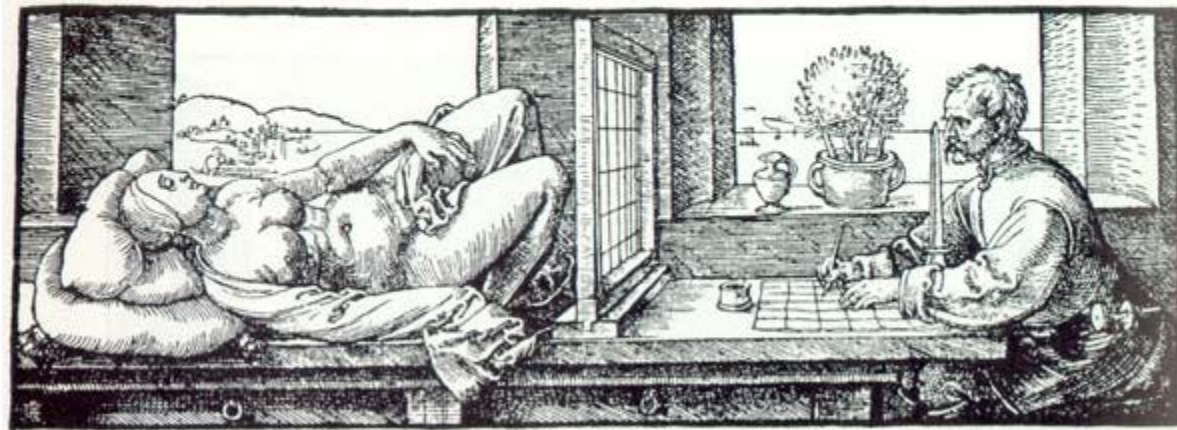


# Practical formulas for light transport

- P. Dutré: **Global Illumination Compendium**,  
<http://people.cs.kuleuven.be/~philip.dutre/GI/>

## Global Illumination Compendium

### The Concise Guide to Global Illumination Algorithms



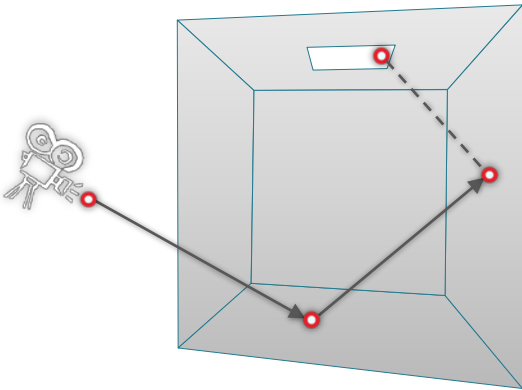
Albrecht Dürer, *Underweysung der Messung mit dem Zirkel und Richtscheyt* (Nuremberg, 1525), Book 3, figure 67.

# **ROBUST GI CALCULATION**

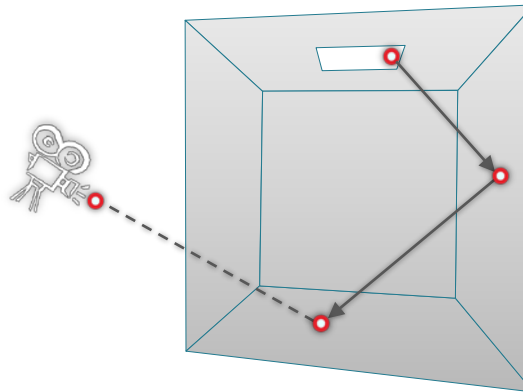


# Path sampling techniques

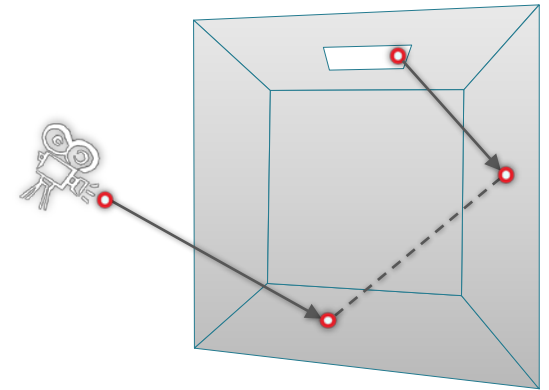
## Path tracing



## Light tracing



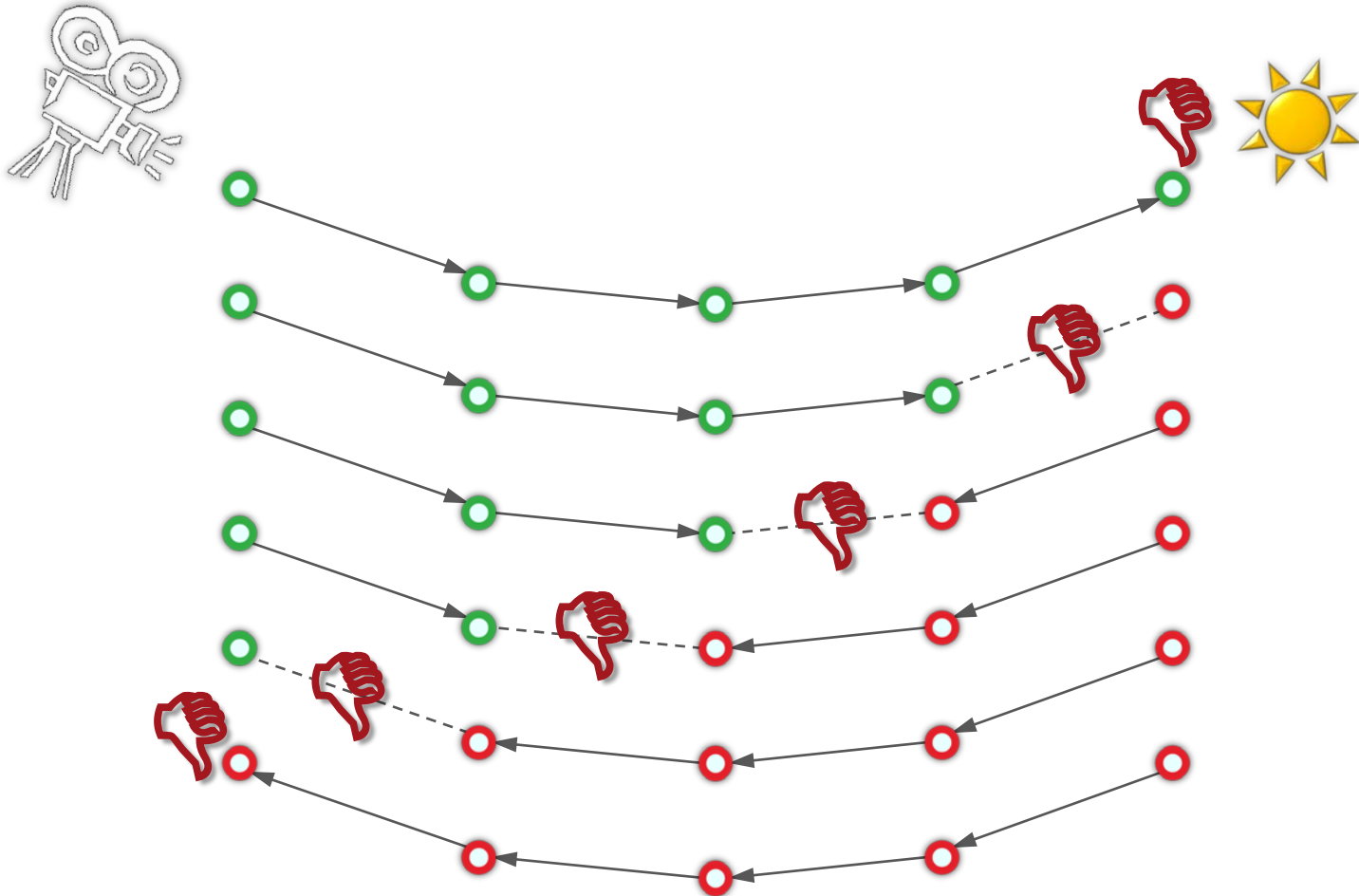
## Bidirectional path tracing



# All possible bidirectional techniques

○ vertex on a **light sub-path**

○ vertex on an **eye subpath**

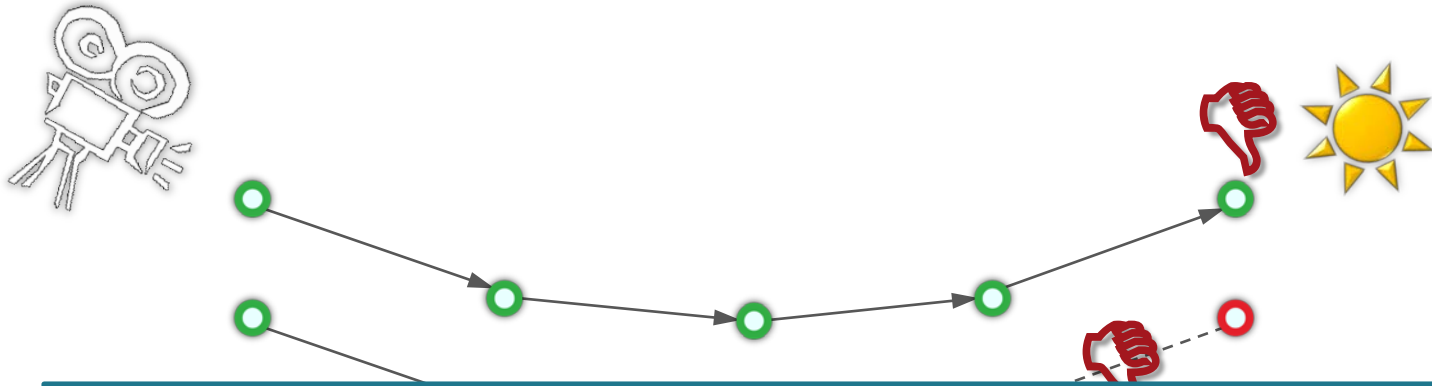




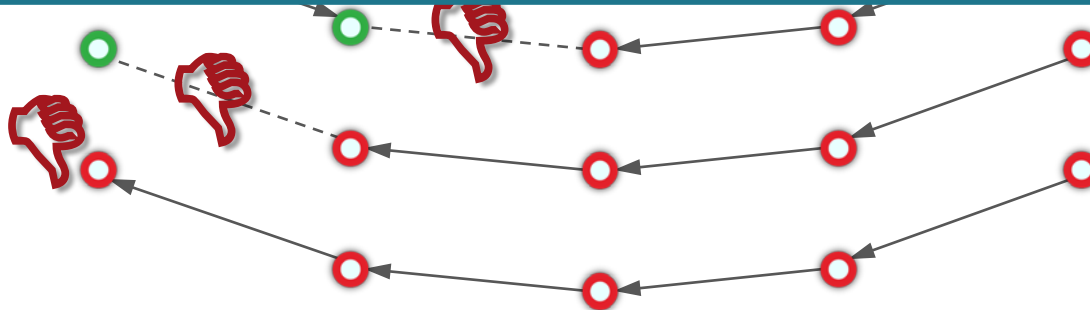
# All possible bidirectional techniques

○ vertex on a **light sub-path**

○ vertex on an **eye subpath**



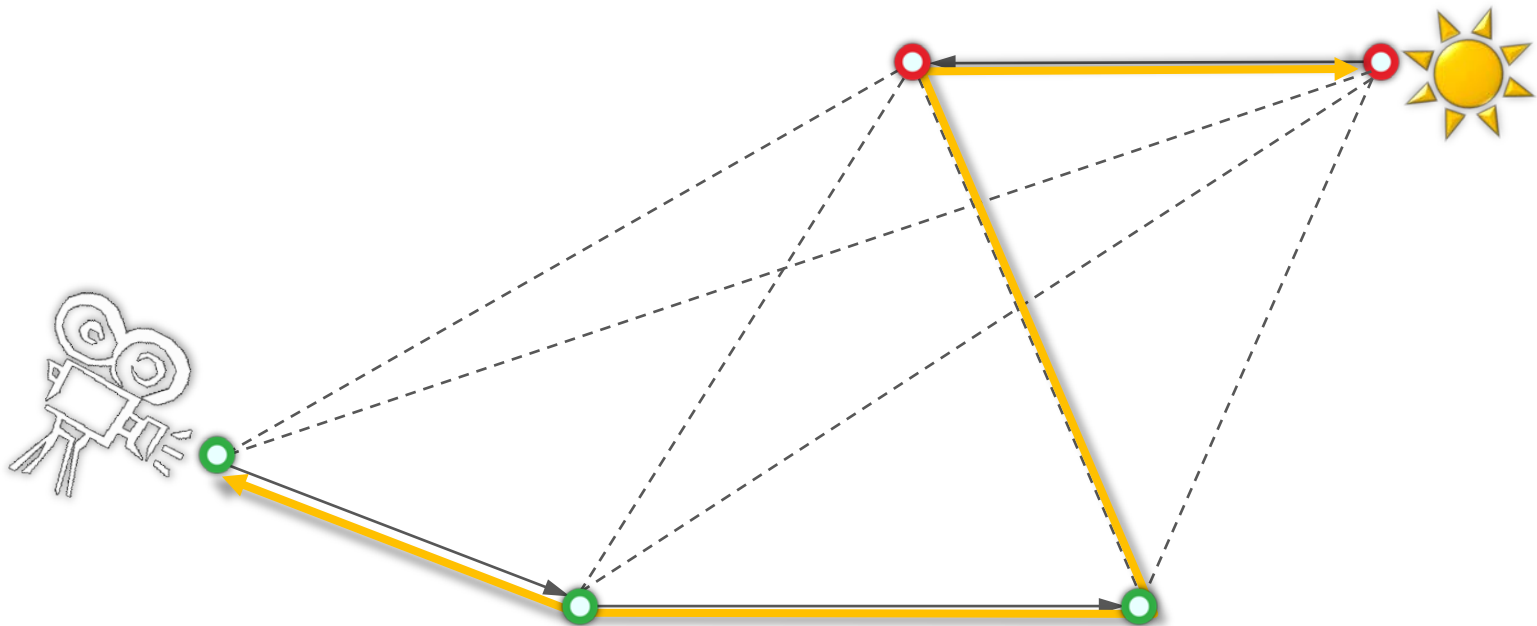
no single techniques importance  
samples all the terms



# Bidirectional path tracing

- Use **all** of the above sampling techniques
- Combine using **Multiple Importance Sampling**

# BPT Implementation



# Results



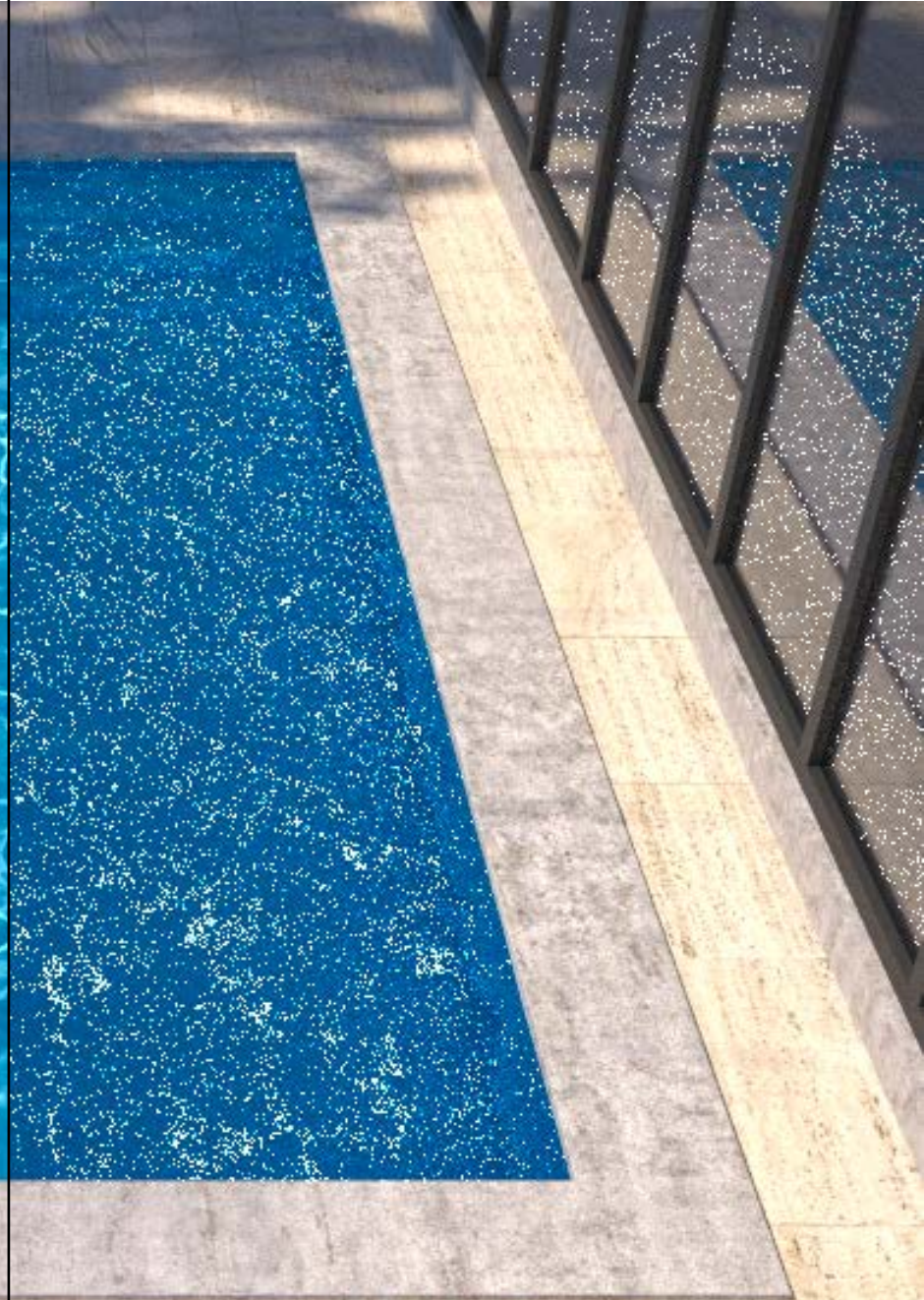
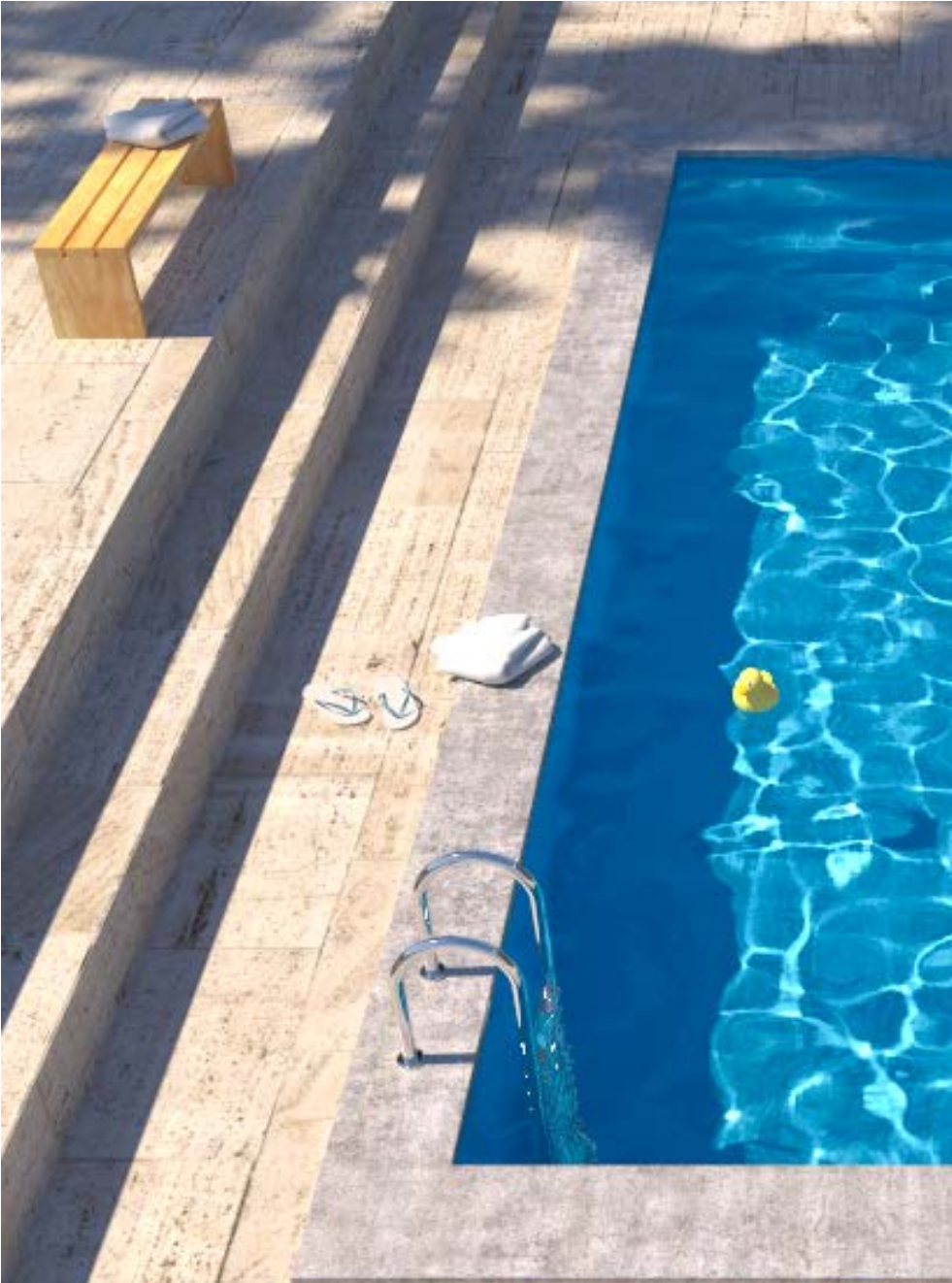
BPT, 25 samples per pixel



PT, 56 samples per pixel

Images: Eric Veach





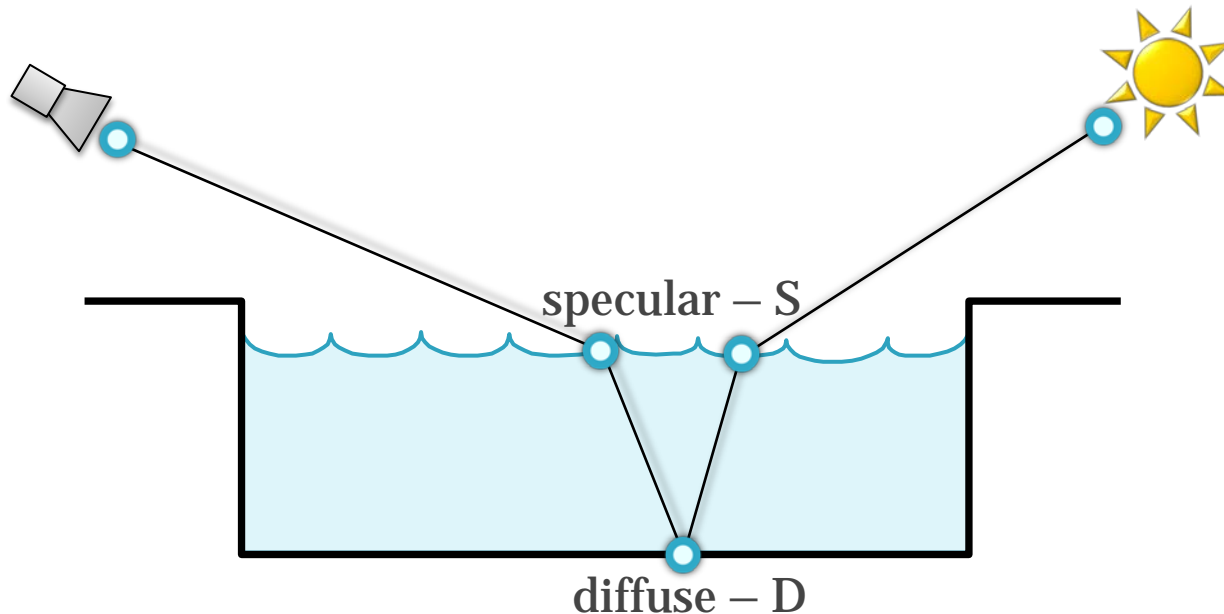
**Reference solution**

**Bidirectional path tracing**



# Insufficient path sampling techniques

- In BPT, **Specular-Diffuse-Specular** paths sampled with zero (or very small) probability



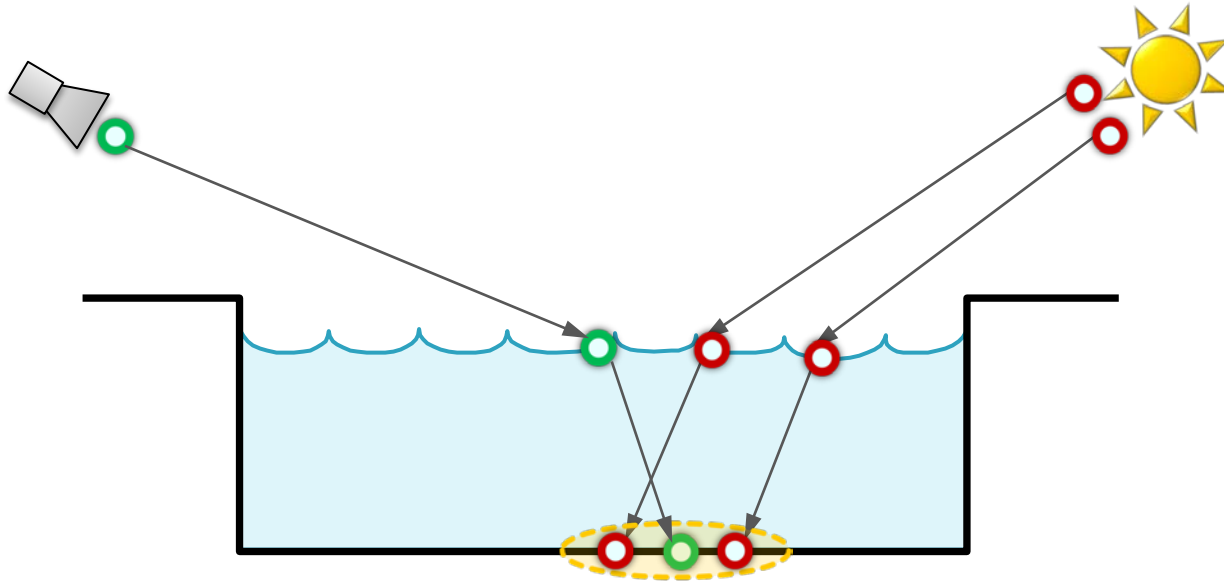
**PHOTON MAPPING**

**(DENSITY ESTIMATION)**



# Photon mapping (Density estimation)

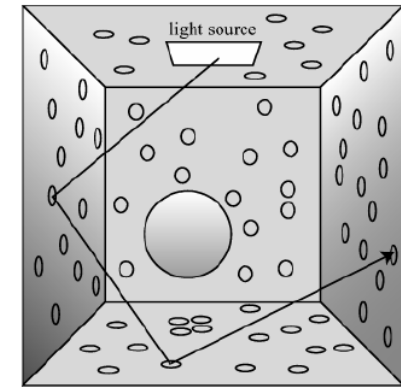
1. Many fwd walks + store collisions (“photon map”)
2. Radiance estimate: (Kernel) **density estimation**



# Photon mapping

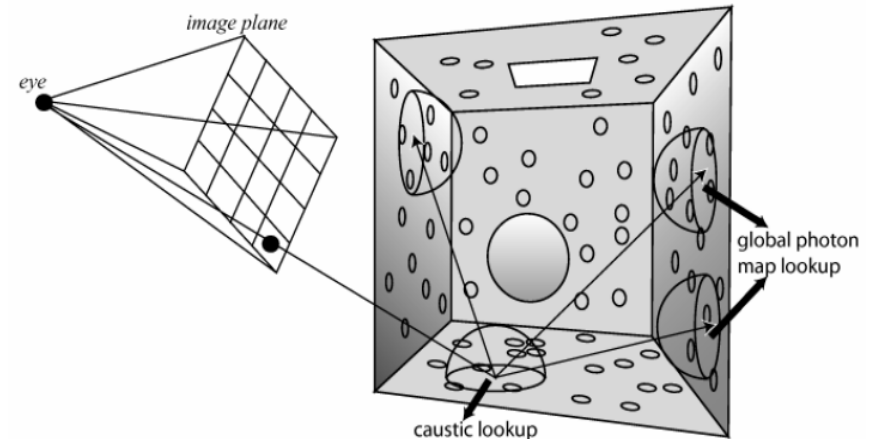
## 1. Photon tracing

- ❑ „Photon“ emission
- ❑ Particle tracing
- ❑ Storage in the „photon map“



## 2. Rendering with photon map

- ❑ Like path tracing
- ❑ Photon map query instead of recursion



# Photon mapping – SDS paths



© H.W.Jensen



© Wojciech Jarosz



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**Our work:**

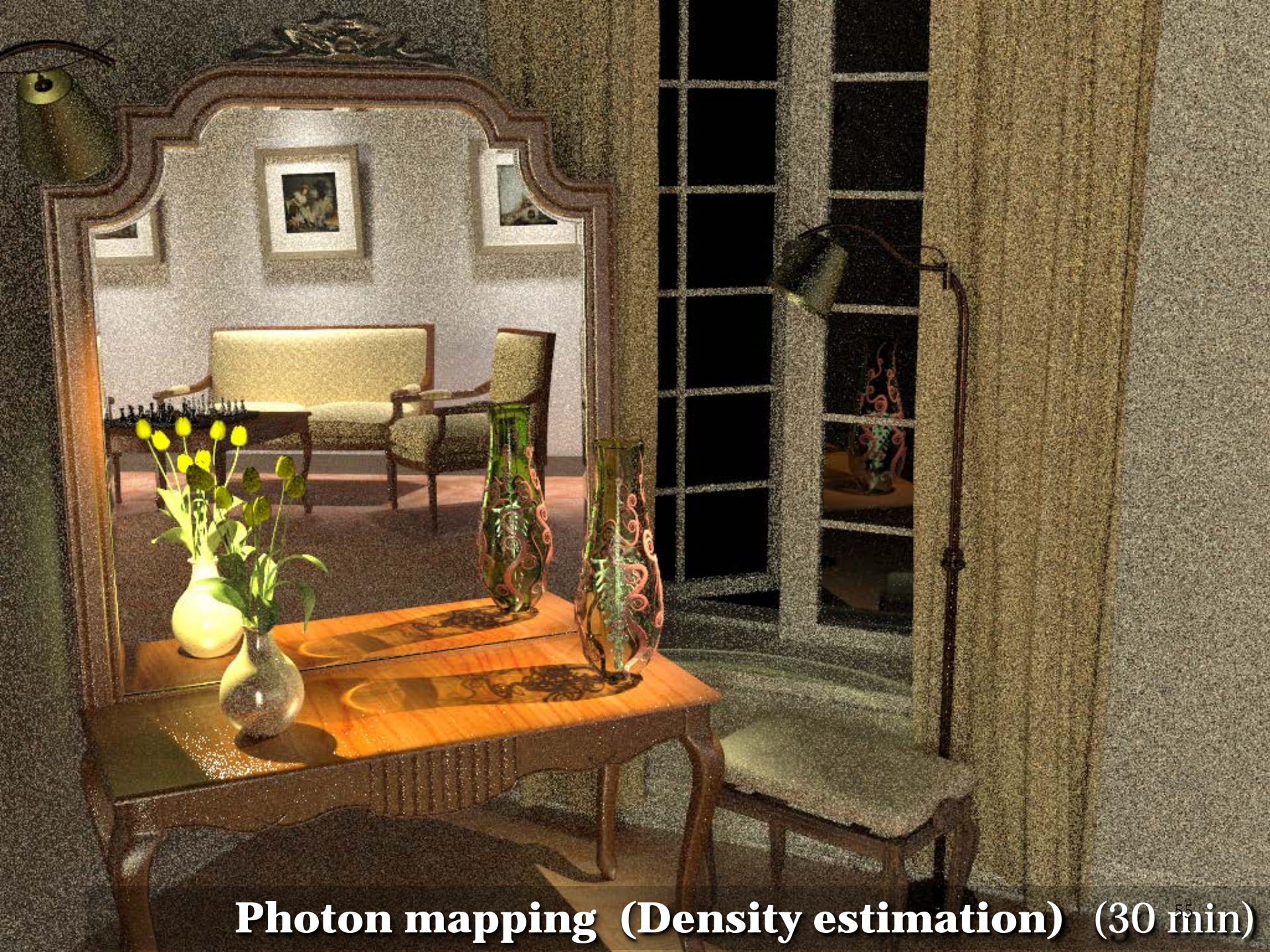
**Vertex Connection and Merging  
BPT + PM combination**

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**Bidirectional path tracing (30 min)** <sup>54</sup>





**Photon mapping (Density estimation) (30 min)**





**Vertex connection and merging (30 min)** <sup>56</sup>

# THANK YOU!

## Questions?



Computer  
Graphics  
Charles  
University

[\[cgg.mff.cuni.cz/~jaroslav\]](http://cgg.mff.cuni.cz/~jaroslav)